Properties of the Tensor Mesons $f_2(1270)$ and $f_2'(1525)$

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Abstract

In the $f_2(1270)-f_2'(1525)$ mixing framework, the isoscalar singlet-octet mixing angle for $1\ ^3P_2$ tensor nonet is determined to the value of 27.5° and the decays of the two states are investigated. Comparing the predicted results of the decays of the two states with the available experimental data, we find that the predicted results are in good agreement with the experimental data, which shows that the $f_2(1270)$ and $f_2'(1525)$ wave functions don't need other components such as glueballs or the $2\ ^3P_2$, $3\ ^3P_2$, ... $q\overline{q}$.

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1. INTRODUCTION

In the 1200∼1800 MeV mass range, one expects that a tensor glueball, the $1\ ^3P_2$, $2\ ^3P_2$, $3\ ^3P_2$ and $1\ ^3F_2$ nonets exist. At present, in this mass range, 13 isoscalar tensor states are claimed to exist experimentally [1]. The state $f_2(1430)$ was claimed to be found in the data on the double-Pomeron-exchange reaction $pp \to p_f(\pi^+\pi^-)p_s$ at $\sqrt{s} = 63$ GeV in an experiment R807 at CERN ISR [2], however, recent experiments on the same reaction do not see any evidence for $f_2(1430)$ [3]. Its existence needs further experimental confirmation. Among the other states, $f_2(1270)$ and $f_2'(1525)$ are well known as the ground tensor states. Above $f_2'(1525)$, none of the reported isoscalar tensor states can be definitely assigned to be the member of the $2\ ^3P_2$, $3\ ^3P_2$, $1\ ^3F_2$ nonets or the tensor glueball [4]. Recently, it is controversial that whether $f_2(1270)$ and $f_2'(1525)$ need glueballs components or not. Ref. [5] sifted these overpopulated isoscalar tensor states using Schwinger-type mass relations derived from a mass matrix in which only the $q\overline{q}$-glueball coupling was considered. Inputing the masses of some observed but possibly confused states, Ref. [5] found that the physical tensor mesons $f_2(1270)$ and $f_2'(1525)$ have a substantial glueball content. Ref. [6] assumed that $f_2(1640)/f_2(1710)$ is the quarkonia-glueball mixing state, and investigated the mixing of $f_2(1270)$, $f_2'(1525)$ and $f_2(1640)/f_2(1710)$ in the $1\ ^3P_2$ $N = (u\overline{u} + d\overline{d})/\sqrt{2}$, $1\ ^3P_2$ $S = s\overline{s}$ and $G = gg$ basis. Ref. [6] suggested that the absence of the gluonic components in the tensor mesons $f_2(1270)$ and $f_2'(1525)$ due to the predicted branching ratios are incompatible with the experimental data. We favor the suggestion that $f_2(1270)$ and $f_2'(1525)$ don't need other components such as glueballs. Since the mass of the lowest lying tensor glueball predicted by lattice QCD is larger than 2 GeV [7], which is far from the masses of $f_2(1270)$ and $f_2'(1525)$, one can qualitatively expect that the mixing between the tensor glueball and the $1\ ^3P_2$ $q\overline{q}$ would be rather little [8]. However, we propose that the states chosen in Ref. [6] are too arbitrary. First, the spin of $f_J(1710)$ has been controversial [9]. Close *et al.* argued that $f_J(1710)$ would be a $q\overline{q}$ state if $J = 2$ and $f_J(1710)$ would be a mixed $q\overline{q}$ glueball having a large glueball component if $J = 0$ [10]. However, evidence for spin 0 have accumulated

recently in all production modes for $f_J(1710)$ [3,11], and the state $f_J(1710)$ with $J=0$ has been cited by Particle Data Group 2000 (PDG 2000) [4]. Second, there is not any evidence that $f_2(1640)$ has advantages over other states to be assigned as a tensor glueball mixing with $1\ ^3P_2\ q\bar{q}$. In the viewpoint of A.V. Anisovich *et al.* [12], it seems reasonable to assign $f_2(1640)$ as the first excitation of $f_2(1270)$ and $f_2(1810)$ as the first excitation of $f_2'(1525)$. If so, it is obviously unreasonable to discuss the mixing of $f_2(1270)$, $f_2'(1525)$ and $f_2(1640)$ in the $1\ ^3P_2\ N$, $1\ ^3P_2\ S$ and G basis. Third, according to the masses of the states chosen in Ref. [6], the mass of the lowest lying tensor glueball is determined to be about 1.5 GeV. Such a low mass tensor glueball would be very difficultly accommodated by lattice QCD which predicts the mass of the tensor glueball is larger than 2 GeV [7]. Finally, as mentioned above, except for $f_2(1270)$ and $f_2'(1525)$, none of the reported isoscalar tensor states can be definitely assigned to be the $2\ ^3P_2$, $3\ ^3P_2$, $1\ ^3F_2\ q\bar{q}$ or the tensor glueball. There thus are not any convincing reasons to only choose $f_2(1640)/f_2(1710)$ but not other state to mix with $f_2(1270)$ and $f_2'(1525)$.

In this work, we shall avoid all the isoscalar tensor states which are confused or need further experimental confirmation, and adopt a simple model to quantitatively check that whether the $f_2(1270)$ and $f_2'(1525)$ wave functions need other components such as glueballs or the $1\ ^3P_2$, $2\ ^3P_2$, $3\ ^3P_2$, ... $q\bar{q}$ or not.

2. MIXING MODEL

In the $1\ ^3P_2\ |N\rangle = |u\bar{u}+d\bar{d}\rangle/\sqrt{2}$, $1\ ^3P_2\ |S\rangle = |s\bar{s}\rangle$ basis, the mass-squared matrix describing the quarkonia-quarkonia mixing can be written as follows [13]:

$$M^2 = \begin{pmatrix} M_N^2 + RA & \sqrt{R}A \\ \sqrt{R}A & M_S^2 + A \end{pmatrix}, \tag{1}$$

where M_N and M_S are the masses of the bare states $1\ ^3P_2\ |N\rangle$ and $1\ ^3P_2\ |S\rangle$, respectively; A is a mixing parameter which describes the transition amplitude of $s\bar{s}$ annihilation and reconstitution via intermediate gluons states. The appearance of R means that we consider

the possibility that the transition between $q\overline{q}$ and $q'\overline{q'}$ is flavor-dependent. Here we assume that the physical states $|f_2(1270)\rangle$ and $|f_2'(1525)\rangle$ are the eigenvectors of the matrix M^2 with the eigenvalues of $M^2_{f_2(1270)}$ and $M^2_{f_2'(1525)}$, respectively. Diagonalizing the mass matrix M^2, we have

$$UM^2U^\dagger = \begin{pmatrix} M^2_{f_2(1270)} & 0 \\ 0 & M^2_{f_2'(1525)} \end{pmatrix}, \tag{2}$$

the physical states $|f_2(1270)\rangle$ and $|f_2'(1525)\rangle$ can be written as

$$\begin{pmatrix} |f_2(1270)\rangle \\ |f_2'(1525)\rangle \end{pmatrix} = U \begin{pmatrix} |N\rangle \\ |S\rangle \end{pmatrix}, \tag{3}$$

where the unitary matrix U can be given by

$$\begin{pmatrix} X_{f_2(1270)} & Y_{f_2(1270)} \\ X_{f_2'(1525)} & Y_{f_2'(1525)} \end{pmatrix} = \begin{pmatrix} \sqrt{R}A/C_1 & (M^2_{f_2(1270)} - M^2_N - RA)/C_1 \\ \sqrt{R}A/C_2 & (M^2_{f_2'(1525)} - M^2_N - RA)/C_1 \end{pmatrix} \tag{4}$$

with $C_1 = \sqrt{RA^2 + (M^2_{f_2(1270)} - M^2_N - RA)^2}$, $C_2 = -\sqrt{RA^2 + (M^2_{f_2'(1525)} - M^2_N - RA)^2}$. It follows from Eqs. (1) and (2) that

$$M^2_N + M^2_S + RA + A = M^2_{f_2(1270)} + M^2_{f_2'(1525)}, \tag{5}$$

$$(M^2_N + RA)(M^2_S + A) - RA^2 = M^2_{f_2(1270)} M^2_{f_2'(1525)}. \tag{6}$$

From Eqs. (5) and (6), A and R can be derived as

$$A = \frac{(M^2_{f_2(1270)} - M^2_S)(M^2_S - M^2_{f_2'(1525)})}{M^2_S - M^2_N}, \tag{7}$$

$$R = \frac{(M^2_{f_2(1270)} - M^2_N)(M^2_N - M^2_{f_2'(1525)})}{M^2_{f_2(1270)} - M^2_S)(M^2_{f_2'(1525)} - M^2_S)}. \tag{8}$$

Apart from $M_{f_2(1270)} = 1.2754$ GeV and $M_{f_2'(1525)} = 1.525$ GeV [4], we take $M_N = M_{a_2(1320)} = 1.318$ GeV and $M_S = 1.55$ GeV [5] as input, the numerical form of the unitary matrix can be given by

$$U = \begin{pmatrix} X_{f_2(1270)} & Y_{f_2(1270)} \\ X_{f_2'(1525)} & Y_{f_2'(1525)} \end{pmatrix} = \begin{pmatrix} -0.991 & -0.135 \\ 0.135 & -0.991 \end{pmatrix}, \tag{9}$$

then the physical states $|f_2(1270)\rangle$ and $|f_2'(1525)\rangle$ can be given by

$$|f_2(1270)\rangle = -0.991|N\rangle - 0.135|S\rangle, \tag{10}$$

$$|f_2'(1525)\rangle = 0.135|N\rangle - 0.991|S\rangle. \tag{11}$$

If we re-express the two physical states in the Gell-Mann $|8\rangle = |u\overline{u} + d\overline{d} - 2s\overline{s}\rangle/\sqrt{6}$, $|1\rangle = |u\overline{u} + d\overline{d} + s\overline{s}\rangle/\sqrt{3}$, $|f_2(1270)\rangle$ and $|f_2'(1525)\rangle$ can be read as

$$|f_2'(1525)\rangle = \cos\theta_T|8\rangle - \sin\theta_T|1\rangle, \tag{12}$$

$$|f_2(1270)\rangle = \sin\theta_T|8\rangle + \cos\theta_T|1\rangle, \tag{13}$$

with $\theta_T = 27.5^\circ$, which is in good agreement with the value of 28° given by PDG 2000 [4].

3. DECAYS

For the hadronic decays of $f_2(1270)$ and $f_2'(1525)$ into two pseudoscalar mesons, we consider the coupling modes as indicated in Fig. I: i) the direct coupling of the quarkonia components of the initial mesons to the final pseudoscalar mesons occurring as the leading order decay mechanism, and ii) the coupling of the quarkonia of the initial mesons to the final pseudoscalar mesons via intermediate gluons states occurring as the next leading order decay mechanism. Based on these coupling modes, the effective Hamiltonian describing the hadronic decays of $f_2(1270)$ and $f_2'(1525)$ into two pseudoscalar mesons can be described as [14]

$$H_{eff} = g_1\mathbf{Tr}(f_F P_F P_F) + g_2\mathbf{Tr}(f_F)\mathbf{Tr}(P_F P_F), \tag{14}$$

where g_1 and g_2 describe the effective coupling strengths of the coupling modes i) and ii), respectively; f_F and P_F are 3×3 flavor matrixes describing the $q\overline{q}$ components of the initial tensor mesons and the final pseudoscalar mesons, respectively. Based on the mixing scheme mentioned in section 2, f_F can written as

$$f_F = \begin{pmatrix} \sum_i \frac{X_i}{\sqrt{2}} i & 0 & 0 \\ 0 & \sum_i \frac{X_i}{\sqrt{2}} i & 0 \\ 0 & 0 & \sum_i Y_i i \end{pmatrix} \quad (i = f_2(1270),\ f_2'(1525)). \tag{15}$$

P_F can be written as

$$P_F = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & K^0 \\ K^- & \overline{K^0} & -\sqrt{2}\beta\eta + \sqrt{2}\alpha\eta' \end{pmatrix}, \tag{16}$$

where

$$\alpha = (\cos\theta_p - \sqrt{2}\sin\theta_p)/\sqrt{6}, \quad \beta = (\sin\theta_p + \sqrt{2}\cos\theta_p)/\sqrt{6}, \tag{17}$$

θ_p is the singlet-octet mixing angle for pseudoscalar nonet, here we take $\theta_p = -15.5^\circ$ [15]. Introducing $r_1 = g_2/g_1$, from Eqs. (14), (15) and (16), we have

$$\frac{\Gamma(i \to \pi\pi)}{\Gamma(i \to K\overline{K})} = 3\left(\frac{q_{i\pi\pi}}{q_{iK\overline{K}}}\right)^5 \frac{[X_i + (2X_i + \sqrt{2}Y_i)r_1]^2}{[X_i + \sqrt{2}Y_i + (4X_i + 2\sqrt{2}Y_i)r_1]^2}, \tag{18}$$

$$\frac{\Gamma(i \to \eta\eta)}{\Gamma(i \to K\overline{K})} = 2\left(\frac{q_{i\eta\eta}}{q_{iK\overline{K}}}\right)^5 \frac{[\sqrt{2}\alpha^2 X_i + 2\beta^2 Y_i + (\sqrt{2}X_i + Y_i)r_1]^2}{[X_i + \sqrt{2}Y_i + (4X_i + 2\sqrt{2}Y_i)r_1]^2}, \tag{19}$$

where $i = f_2(1270),\ f_2'(1525)$, $q_{iP_1P_2}$ is the decay momentum for the decay mode $i \to P_1P_2$,

$$q_{iP_1P_2} = \sqrt{M_i^2 - (M_{P_1} + M_{P_2})^2}\sqrt{M_i^2 - (M_{P_1} - M_{P_2})^2}, \tag{20}$$

M_{P_1} and M_{P_2} are the masses of the final pseudoscalar mesons P_1 and P_2, respectively, and we take $M_K = \sqrt{(M_{K^\pm}^2 + M_{K^0}^2)/2}$.

For the two-photon decays of $f_2(1270)$ and $f_2'(1525)$, we have [16]

$$\frac{\Gamma(i \to \gamma\gamma)}{\Gamma(a_2 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{M_i}{M_{a_2}}\right)^3 (5X_i + \sqrt{2}Y_i)^2, \tag{21}$$

with $i = f_2(1270),\ f_2'(1525)$.

5. COMPARISON WITH THE EXPERIMENTAL DATA

Now we wish to compared the theoretical results of Eqs. (18), (19) and (21) with the available experimental data. From Eq. (9), the theoretical results of Eq. (21) can be directly obtained as shown in Table I. In order to obtain the theoretical results of Eqs. (18) and (19), we should first determine the value of the unknown parameter r_1. In this procedure, we take 0.0092, the experimental datum of $\frac{\Gamma(f_2'(1525)\to\pi\pi)}{\Gamma(f_2'(1525)\to K\overline{K})}$ as input, and determine the parameter r_1 to be the value of 0.082 or 0.161 by solving the equation $\frac{\Gamma(f_2'(1525)\to\pi\pi)}{\Gamma(f_2'(1525)\to K\overline{K})} = 0.0092$. Using Eq. (9) and the value of r_1, the theoretical results of Eqs. (18) and (19) are determined as shown in Table I.

From Table I, we find the theoretical results are in good agreement with the experimental data, especially for $r_1 = 0.082$, i.e., the present experimental data support $|f_2(1270)\rangle = -0.991|N\rangle - 0.135|S\rangle$ and $|f_2'(1525)\rangle = 0.135|N\rangle - 0.991|S\rangle$, which shows that the $f_2(1270)$ and $f_2'(1525)$ wave functions don't need other components such as glueballs or the $2\ ^3P_2$, $3\ ^3P_2$, ... $q\overline{q}$.

6. SUMMARY AND CONCLUSIONS

Under the two-state mixing scheme, we determined the quarkonia content of $f_2(1270)$ and $f_2'(1525)$, and investigate the decays of the two states. The predicted results are in good agreement with the experimental data. Our conclusions are as follows:

1. The $f_2(1270)$ and $f_2'(1525)$ wave functions don't need other components such as glueballs and the $2\ ^3P_2$, $3\ ^3P_2$, ... $q\overline{q}$.

2. $f_2(1270)$ is a nearly pure $1\ ^3P_2$ $(u\overline{u}+d\overline{d})/\sqrt{2}$ state ($\sim 98.2\%$) and $f_2'(1525)$ is a nearly pure $1\ ^3P_2$ $s\overline{s}$ state ($\sim 98.2\%$). The isoscalar singlet-octet mixing angle for $1\ ^3P_2$ tensor nonet is determined to be the value of $27.5°$.

6. ACKNOWLEDGMENTS

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TABLES

Decay	Exp. [4]	Pred.		Decay	Exp. [4]	Pred.	
		$r_1 = 0.082$	$r_1 = 0.161$			$r_1 = 0.082$	$r_1 = 0.161$
$\frac{\Gamma(f_2 \to \gamma\gamma)}{\Gamma(a_2 \to \gamma\gamma)}$	2.59 ± 0.60	2.67	2.67	$\frac{\Gamma(f_2' \to \gamma\gamma)}{\Gamma(a_2 \to \gamma\gamma)}$	0.10 ± 0.04	0.09	0.09
$\frac{\Gamma(f_2 \to \pi\pi)}{\Gamma(f_2 \to K\overline{K})}$	18.41 ± 2.52	15.67	13.78	$\frac{\Gamma(f_2' \to \pi\pi)}{\Gamma(f_2' \to K\overline{K})}$	0.0092 ± 0.0018	0.0092^*	0.0092^*
$\frac{\Gamma(f_2 \to \eta\eta)}{\Gamma(f_2 \to K\overline{K})}$	0.10 ± 0.03	0.11	0.11	$\frac{\Gamma(f_2' \to \eta\eta)}{\Gamma(f_2' \to K\overline{K})}$	0.12 ± 0.04	0.10	0.11

TABLE I. The predicted and experimental results about the decays of $f_2(1270)$ and $f_2'(1525)$. f_2 and f_2' respectively denote $f_2(1270)$ and $f_2'(1525)$. (* input).

FIGURES

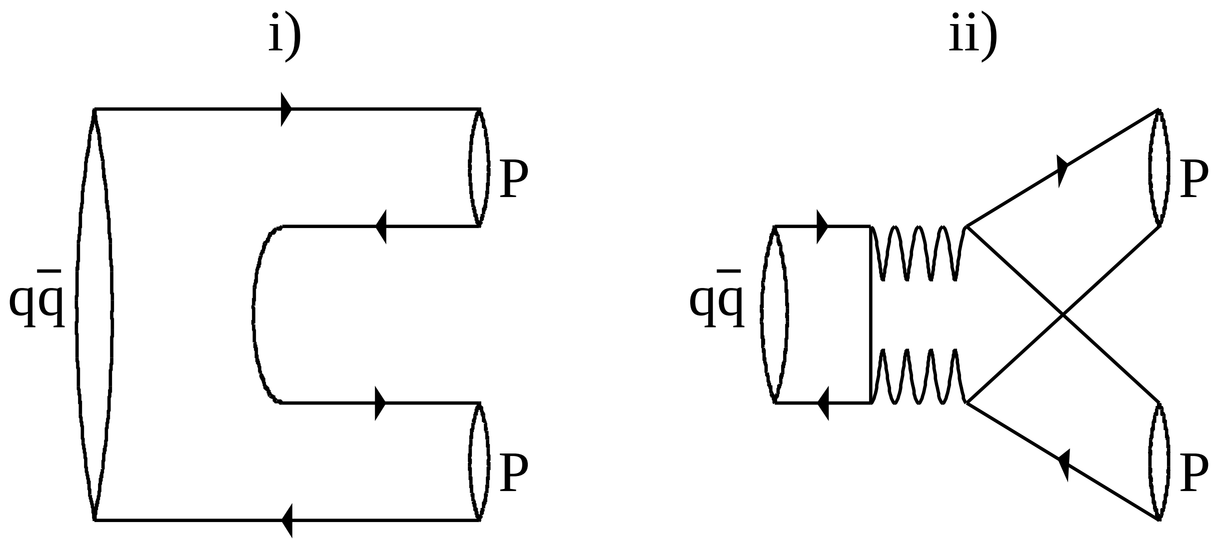

FIG. 1. The coupling modes of the quarkonia of $f_2(1270)$ and $f_2'(1525)$ to the pseudoscalar meson pairs (PP) considered in this work. i) The direct coupling of the quarkonia components of the initial mesons to the final pseudoscalar mesons occurring as the leading decay mechanism. ii) The coupling of the quarkonia components of the initial mesons to the final pseudoscalar mesons via intermediate gluons states occurring as the next leading order decay mechanism.